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Numerical Estimation in Children for Both Positive and Negative Numbers

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Numerical estimation has been used to study how children mentally represent numbers for many years (e.g., Siegler & Opfer, 2003). However, these studies have always presented children with positive numbers and positive number lines. Children's mental representation of negative numbers has never been addressed. The present study tested children in the 2nd, 4th, and 6th grades to assess their mental representations of both positive and negative numbers using a standard numerical estimation task. We replicated the shift from a logarithmic to linear representation for positive numbers (0–1,000 scale) in that 2nd graders represented positive numbers logarithmically, but 4th and 6th graders represented the numbers linearly. Furthermore, children's representation of negative numbers paralleled their representations of positive numbers and showed the same shift from a logarithmic representation at Grade 2 to linear representations at Grades 4 and 6. This is the first study to provide data on children's representation of negative numbers, and the implications of these findings are discussed.

Being able to mentally represent numbers is an important and necessary skill for many of our daily tasks that require processing numbers. Researchers have been investigating how children and adults mentally represent numbers for many years (see Booth & Siegler, 2006; Gevers et al., 2010; Siegler & Opfer, 2003). While various theories have been proposed to explain how children mentally represent numbers (see Friso-van den Bos, Kolkman, Kroesbergen, & Leseman, 2014, for a good overview), Siegler and colleagues' theory is one of the most common approaches for explaining children's mental representation of numbers (Booth & Siegler, 2006; Siegler & Opfer, 2003; Siegler, Thompson, & Opfer, 2009). Siegler and colleagues have proposed that children use a variety of representations (both linear and logarithmic) to mentally represent numbers, but their representations shift from logarithmic to linear as they age. However, the shift from a logarithmic representation to a linear representation occurs at different ages depending on the scale being presented (Siegler et al., 2009). For example, the shift occurs between kindergarten and second grade for a 0-to-100 scale, but it occurs between third and sixth grade for a 0-to-100,000 scale. Understanding this developmental trajectory is important

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for children's success in school as previous studies have shown that children's performance on these types of estimation tasks relates to their math achievement (Booth & Siegler, 2006).

These studies typically use numerical estimation tasks using a visual number line to evaluate children's mental representations. While numerical estimation tasks can take several forms (e.g., estimating dots or objects), we will use the term "numerical estimation" to refer to those tasks using estimation of numbers on a visual number line for the purposes of this article. Numerical estimation tasks can either require children to make a mark on a number line indicating where a given number should fall (i.e., number-to-position or NP tasks) or require children to come up with a guess for what number is marked on a number line (i.e., position-to-number task; Siegler & Opfer, 2003). Despite the plethora of studies demonstrating how children mentally represent positive numbers, how children mentally represent negative numbers remains an unanswered question. All previous studies with numerical estimation tasks involving children have used number lines that begin at 0 and extend to a positive value (e.g., 100 or 1,000).

Recently, Barth and colleagues (Barth & Paladino, 2011; Slusser, Santiago, & Barth, 2013; Sullivan, Juhasz, Slattery, & Barth, 2011) provided evidence that challenges the theory that children's representation of numbers shifts from a logarithmic to linear representation. Rather, they suggested that children's (and adults') pattern of responding on number estimation tasks is best fit by a proportion judgment model. The proportion judgment model suggests that children divide the number line into segments and use those "landmarks" as a basis for judging where each number should fall on the number line. Furthermore, some researchers (Cohen & Sarnecka, 2014) have argued that number estimation tasks do not reflect internal representations of number, but rather reflect children's measurement skills. These counterarguments raise interesting questions about how children represent numbers and the strategies that they use to solve number estimation tasks. However, like all previous number representation research, these studies only focus on how children represent positive numbers. It is still unknown how children may solve these tasks using negative numbers.

Although research on children's mental representation of negative numbers is lacking, various studies exist that investigate how adults may mentally represent negative numbers (see Tzelgov, Ganor-Stern, & Maymon-Schreiber, 2009, and Varma & Schwartz, 2011). For example, do adults represent negative numbers holistically (polarity and magnitude are stored together) or individually (polarity and magnitude are represented as separate components)? Some evidence with adults exists to suggest that negative numbers are processed in component parts (Tzelgov et al., 2009, but see Ganor-Stern, Pinhas, Kallai, & Tzelgov, 2010). Tzelgov et al. (2009) found that adults represent the polarity of a number (i.e., sign of the number, positive or negative) separately from the magnitude (i.e., value). Studies have also investigated whether adults use a full mental number line (positive and negative represented together) or whether they use a positive number line but use short cuts or strategies to transpose negative numbers (see Karjcsi & Igacs, 2010; Varma & Schwartz, 2011). Some evidence suggests that adults represent all numbers on a single number line (positive numbers) and simply use a transformation by adding the negative sign when dealing with negative numbers (Karjcsi & Igacs, 2010). From a developmental perspective, it is interesting to consider whether children's mental representations of negative numbers differ from those of adults and, if they do, how children's representations of negative numbers differ. Some physiological data suggest that children may use different brain areas than adults when processing negative numbers (Gullick & Wolford, 2013; Varma & Schwartz, 2011). It is possible that children, like adults, will process negative numbers in component parts (polarity and magnitude separately), or they may use different strategies altogether to approach these tasks. While previous findings and the results from the current study can provide preliminary answers to these questions, we hope that our discussion of this topic will stimulate future research to further address these issues.

We believe that investigating children's representations of negative numbers is important for several reasons. First, the number line extends from 0 to both positive and negative numbers. By only focusing on positive numbers, we are failing to assess half of the "conceptual space" that can help us understand how children process numbers. Additionally, it is possible, but unknown at this point, that children's performance on number estimation tasks for negative numbers may be just as predictive of academic achievement as their performance on estimation tasks for positive numbers (Booth & Siegler, 2006). If this relationship holds for negative numbers, then the potential exists for research investigating this relationship as well as for interventions for improving math achievement. Currently, researchers are investigating interventions and teaching methods for introducing children to negative numbers (see Prather & Alibali, 2008; Saxe, Diakow, & Gearhart, 2012; Saxe et al., 2010). Although these studies provide evidence of the potential to teach children about negative numbers and the number line, they fail to provide evidence for the underlying mental representations of negative numbers. We believe it is important to assess and understand the basic questions of how children mentally represent negative numbers and how they solve number estimation tasks with negative numbers. Thus, the present study is the first of its kind to extend number-line estimation tasks with children to negative numbers and to provide an important baseline from which future research in this field can extend.

The primary purpose of this study was the use of a number-line assessment to replicate the logarithmic-to-linear shift pattern in positive number sense explored by Booth and Siegler (2006), as well as to examine the hypothesized parallel logarithmic-to-linear shift using negative numbers. We hypothesize that negative numbers will show the same shift as positive numbers because it is likely that children are using a similar strategy as adults and using the positive number line as a basis for their estimation of both positive and negative numbers. Whether we replicate this shift or not, the findings have implications for our theoretical understanding of children's mental representations of number. If children show a shift from a logarithmic to linear representation, then it may suggest that children a) mentally represent negative numbers in a similar fashion to positive numbers, b) use a strategy similar to adults in which they treat negative numbers as positive and then transpose to negative, or c) have some alternative strategy for solving negative number estimation tasks. If we fail to replicate this shift with negative numbers, then it may suggest that children a) do not have a mental representation of negative numbers, or b) mentally represent negative numbers in a different manner than they do positive numbers.

Booth and Siegler (2006) focused on kindergarteners and first, second, third, and fourth graders. We chose to extend Booth and Siegler's study to also examine upper elementary school students (Grade 6). Including older students is important as one goal of this study was to examine the hypothesized presence of the log-to-linear shift for negative numbers. The inclusion of older students is necessary to incorporate students who have been exposed to negative numbers in their mathematics curriculum and to examine the negative number sense of those younger students who have not yet been formally introduced to negative numbers. Furthermore, we also considered alternative explanations of the development of negative number sense. Based on prior work by Barth and Paladino (2011) and Slusser and colleagues (2013), we also examined the proportion estimation model and its ability to account for estimation patterns in both positive and negative numbers.

METHOD

Participants

Participants included 241 elementary-level students from a Midwestern suburban school district. There were 72 second graders (32 boys and 40 girls; average age = 7;9), 82 fourth graders (42 boys and 40 girls; average age = 9;9), and 87 sixth graders (40 boys, 45 girls, and 2 individuals whose gender was unknown; average age = 11;8). Our sample represented 82% to 92% of each grade level in the school. Even though data on the demographics for the individuals in this sample were not collected, our sample was primarily Caucasian and representative of the schools and the larger district. The elementary school at which we tested has the following racial breakdown: 76.8% Caucasian, 6.9% Hispanic, 1.4% African American, and 14.8% Other. Approximately 5.1% of the students are classified by the state as "economically disadvantaged." The middle school at which we tested has the following racial breakdown: 73.5% Caucasian, 5.8% Hispanic, 5.8% African American, and 15.0% classified as other. Approximately 10.7% of the students are classified as "economically disadvantaged."

Procedure and Materials

This study was part of a larger study designed to address children's understanding of negative numbers. As part of this project, data were collected on children's understanding of negative numbers, children's performance on standardized math assessments, and their mental representation of both positive and negative numbers as assessed through numerical estimation tasks (NP task). Currently, we have only analyzed the data from the numerical estimation task data, and therefore, these are the only data examined in this replication study. The NP task was identical to the task used by Opfer and Siegler (2007) with the exception that we gave children negative-number lines in addition to the positive number lines. A 25-cm line was presented to children with two endpoints marked. Just as in the study by Opfer and Siegler, the positive number line had a 0 endpoint on the left and a 1,000 endpoint on the right. The negative number line had a −1,000 endpoint on the left and a 0 endpoint on the right. Each number line had a target value centered above the line. Children were told to mark where they believed the target value fell on the number line (NP task). We used the same values as Opfer and Siegler, which allowed for better discrimination between logarithmic and linear representations. Positive values included 2, 5, 18, 34, 56, 78, 100, 122, 147,150, 163, 179, 246, 366, 486, 606, 722, 725, 738, 754, 818, and 938. Negative values were exactly the same but were presented with a negative sign. Given the variety of number estimation tasks that exist and the importance of the type of task for this skill (see Cohen & Sarnecka, 2014), we chose to use two separate lines (positive and negative) for two reasons. First, we wanted to directly replicate Siegler's work (Opfer & Siegler, 2007) as closely as possible and thus chose to use the original positive number line and values used in those studies. Secondly, to provide as direct a comparison as possible to those previous studies, we chose to keep everything identical and only switch the polarity of the scale and values for the negative numbers.

The first author (CCB) and one to two research assistants tested the children at the school. The second graders were tested in the cafeteria. Any children without a consent form or who refused assent were sent back to the classroom to work on other activities with their teacher. The fourth graders were tested in the library, and any children not participating in the study were sent back to the classroom. The sixth graders were tested during their regular math period in the math classroom. Any children without a consent form or who did not provide assent sat in the back of the classroom and worked on material that the teacher assigned.

The number line task was explained by telling students that they needed to mark where the number appearing at the top of the page belonged on the number line by drawing a line at that point. The type of number scale (positive or negative numbers) was counterbalanced across children, so half of the children were first presented with the negative number lines (a number line going from –1,000 on the left to 0 on the right), and the other half first had the standard positive number lines (0 on the left and 1,000 on the right). Because the children did not have the same number line (positive or negative), a specific example problem was not given (unlike other studies using this method; see Booth & Siegler, 2006, and Opfer & Siegler, 2007). However, the experimenter explicitly told the students that some of the number lines went from 0 to 1,000 and some went from –1,000 to 0. Students were asked to confirm which one they had by raising their hands. After identifying which number line children had, the experimenter repeated the instructions that students were to mark where the number at the top would go on their number line. Then, each child was given 22 pages with one number line per page to complete. Once children completed the first set of numbers, they were instructed to pull out the second packet, which had the opposite number line from what they had just completed (either positive or negative). Again, the experimenter pointed out that they had the opposite number line of what they just completed and highlighted the differing anchor points (0 to 1,000 or –1,000 to 0). Students were then instructed to complete the 22 pages with the second number-line task. To test for an effect of the order of presentation (negative or positive numbers first), an independent *t* test analysis was conducted and showed that there were not significant differences in estimates based on order of presentation ($p > .05$).

While the positive number lines have been used in many other studies (e.g., Booth & Siegler, 2006, and Siegler & Opfer, 2003), the extension to the negative number lines is novel. We feel confident that the negative number-line task is valid as it correlates to students' knowledge of negative numbers as assessed through a measure containing math problems dealing with negative numbers, $r = –.59$, $p < .001$. (Note that data from the negative number line are measured as percent absolute error, hence the negative correlation with negative number knowledge). This negative number assessment was created by the first two authors for the purposes of this larger study. It contained items that tested students' conceptual understanding of negative numbers (e.g., "What is a negative number?"), students' ordinal knowledge (e.g., "Which number is larger? –4 or 2?"), as well as students' ability to perform simple addition and subtraction facts with negative numbers (e.g., "3 + –2 = _____?"). Students were tested with symbolic representations (like the aforementioned examples) and with problems embedded within an applied context (i.e., word problems such as, "The temperature is 0° F. If the temperature drops by 10°, what is the temperature now?"). Items were scored for correct responses and a total score was obtained for each student. This assessment of negative number knowledge not only correlates with the negative number line task, but it also correlates with generalized math ability as measured through a standardized test of math achievement (Measures of Academic Progress [MAP]), $r = .86$, $p < .001$. The MAP test for primary grades was first used in 2006 and is a computerized adaptive test used in many school systems. The MAP contains multiple choice items in the following domains: number and computation, algebra, geometry, and data. Furthermore, students' MAP scores also correlate with the negative number line task, $r = -.57$, $p < .001$, further validating this as a measure of students' ability to make judgments regarding negative numbers.

RESULTS

Positive Values

First, following the model of Booth and Siegler (2006), we calculated each student's percent of absolute error. This calculation involved subtracting the value requested from the calculated location of the student's mark on the number line; this deviation was then divided by 1,000 (the size of the scale). For example, if a student estimated the location of 606 as 470, the percent of absolute error would be calculated as [(470 – 606)/1,000] or 13.6%. Each student's mean percent of absolute error was calculated, and these values were examined for grade-level differences. We found that accuracy did increase with grade level, $F(2, 238) = 80.28$, $p < .001$, similar to the findings of Booth and Siegler (2006). The mean percent of absolute error was 20% for second graders, 9.1% for fourth graders, and 5.5% for sixth graders. A Games-Howell post-hoc analysis indicated that the differences between all grades were significant ($p < .01$).

Next, median estimates of each positive number by grade level were calculated. Differences between this value and the number predicted by the best-fitting exponential, logarithmic, and linear lines were examined. The exponential function was poorly fit at all grade levels and was not included in any other analyses. Paired-sample *t*-tests were used to investigate differences in the quality of the fit at each grade level and to determine if the age-related change from logarithmic to linear representations described by Booth and Siegler (2006) could be replicated. Graphical representations for each grade level were generated (Figure 1) with both linear and logarithmic fit lines. Second graders' number-line estimates for positive values were better fit by the logarithmic ($R^2 = .90$) function than by the linear function ($R^2 = .84$), $t(20) = 13.58$, $p < .01$. However, fourth graders' number-line estimates were better fit by the linear function ($R^2 = .99$) than by the logarithmic function ($R^2 = .69$), $t(20) = 43.20$, $p < .01$. Similarly, the sixth graders' estimates were also better fit by the linear function ($R^2 = .99$) than the logarithmic function ($R^2 = .65$), $t(20) = 58.92$, $p < .01$.

To make sure these results were not due to averaging of students' responses, we compared the variance accounted for by the best-fitting linear, logarithmic, and exponential functions for each student's estimates. As the number of students whose estimates were best fit by the exponential function was minimal (< 5), only the linear and logarithmic functions were further examined. The type of function that fit the majority of students did vary with age, $\chi^2(2, n = 241) = 67.79$, $p < .001$, $V = .53$. When examining each grade level, the logarithmic function provided the best fit for 55.6% of students in Grade 2 and the linear function provided the best fit for 44.4% of students in Grade 2. In Grade 4, the linear

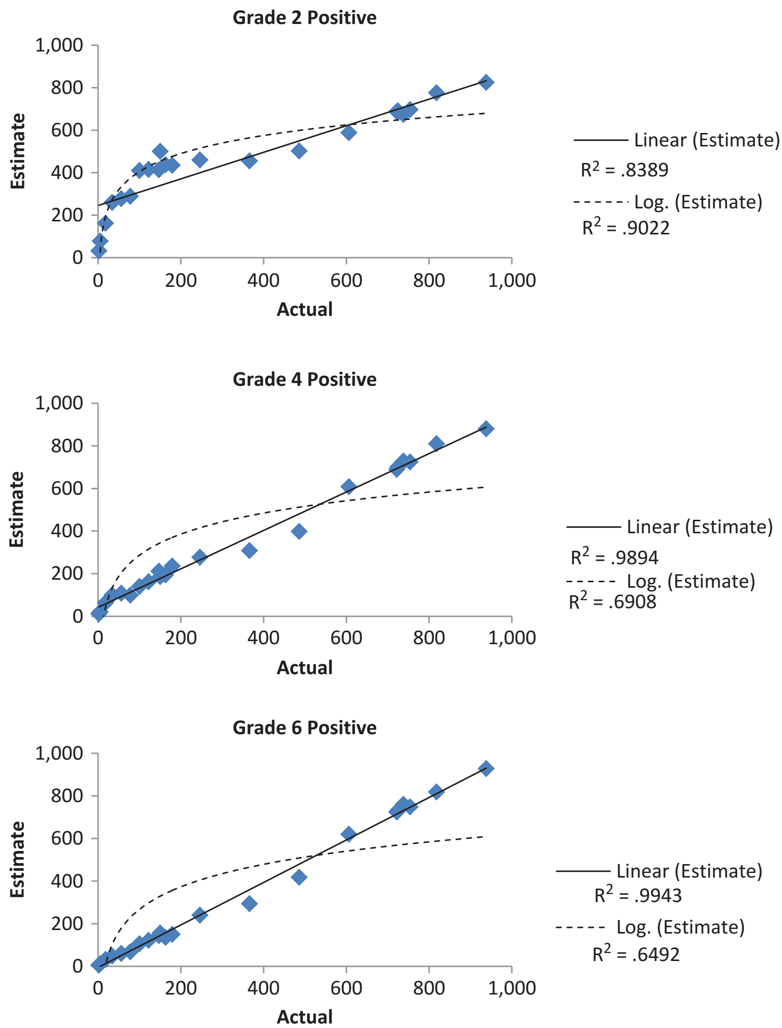

FIGURE 1 Best-fitting equations for median positive number line estimates for second, fourth, and sixth graders. Log. = logarithmic.

function provided the best fit for 81.5% of students and the logarithmic function only provided the best fit for 18.5%. Finally, for students in Grade 6, the linear function provided the best fit for 97.7% of students and the logarithmic provided the best fit for a mere 2.3%. Paired-sample *t*-tests indicated that the fit of the logarithmic (log) function to individual students' estimates was not significantly different than the fit of the linear (lin) function for second graders (mean R^2_{lin} = .57, *SE* = .04, vs. mean R^2_{log} = .59, *SE* = .03), $t(71) = -0.524$, $p > .05$. However, for fourth graders, the fit of the linear function was a significantly better fit than the logarithmic function (mean R^2_{lin} = .88, *SE* = .01, vs. mean R^2_{log} = .68, *SE* =

.01), $t(81) = 9.93$, $p < .001$, $d = 1.1$; likewise, sixth graders followed a similar pattern (mean $R^2_{\text{lin}} = .95$, $SE = .01$, vs. mean $R^2_{\text{log}} = .65$, $SE = .01$), $t(86) = 25.82$, $p < .001$, $d = 2.8$. A one-way analysis of variance (ANOVA) indicated that the fit of the linear function to individual students' estimates increased with grade level, $F(2, 238) = 74.62$, $p < .001$. Additionally, the Games-Howell post-hoc analysis indicated significant differences between all grade levels ($p < .01$).

Similar to Slusser and colleagues (2013), we eliminated data from children producing responses that were uncorrelated or negatively correlated with the presented number in either the positive or negative condition ($n = 27$). Data from these children are impossible to interpret using the proportion shift models. Note that we did not eliminate these children from the models examining the logarithmic-to-linear shift method (similar to Booth & Siegler, 2006) and we note that this is an important difference between the two approaches. We question the potential bias of overcorrecting the data when eliminating these seemingly valid responses from mostly younger children ($n = 20$) who present a less sophisticated representation of numbers. Models included the proportion judgment analyses, the unbounded power function, and the one- and two-cycle version of the proportional power model. Model comparisons were completed using the methodology of Slusser et al. comparing Akaike information criterion (AICc). Differences in AICc provide a goodness-of-fit index, where a smaller AICc indicates superior fit of the curve to the data. The R^2 is also reported as used in the logarithmic-to-linear shift analyses. We examined performance patterns of Grade 2, Grade 4, and Grade 6 students separately. For Grade 2 students, the unbounded power function ($R^2 = .94$) provides the best fit to the data, while the two-cycle model is the best option for the Grade 4 ($R^2 = .99$) and Grade 6 ($R^2 = .99$) samples. Graphical presentations of the proportion shift analyses are presented in Figure 2.

Negative Values

The same series of data analyses were carried out with students' negative number estimates. We again calculated each student's percent of absolute error and examined grade-level differences. The ANOVA with mean percent absolute error indicated that accuracy again significantly increased with grade level, $F(2, 238) = 51.36$, $p < .001$. A Games-Howell post-hoc analysis indicated that all grade differences were significant ($p < .001$). The mean percent of absolute error was 23.3% for second graders, 13.7% for fourth graders, and 6.5% for sixth graders.

When examining the negative number line estimates made by each grade level, the patterns found with the positive values were again repeated. As shown in Figure 3, second graders' number line estimates for the negative values were better fit by the logarithmic ($R^2 = .92$) function than by the linear function ($R^2 = .80$), $t(20) = 15.16$, $p < .01$. As before, fourth graders' number line estimates were better fit by the linear function ($R^2 = .98$) than by the logarithmic function ($R^2 = .73$), $t(20) = 28.24$, $p < .01$, as were the sixth graders' number line estimates ($R^2 =$ .99 vs. .65), $t(20) = 44.95$, $p < .01$.

As with the positive numbers, to make sure these results were not due to averaging of students' responses, we compared the variance accounted for by the best-fitting linear, logarithmic, and exponential functions for each student's negative number estimates. As the number of students whose estimates were best fit by the exponential function was minimal (< 5), only the linear and logarithmic functions were further examined. The type of function that fit the majority of students did again vary with age, $\chi^2(2, n = 241) = 66.50$, $p < .001$, $V = .53$. When examining

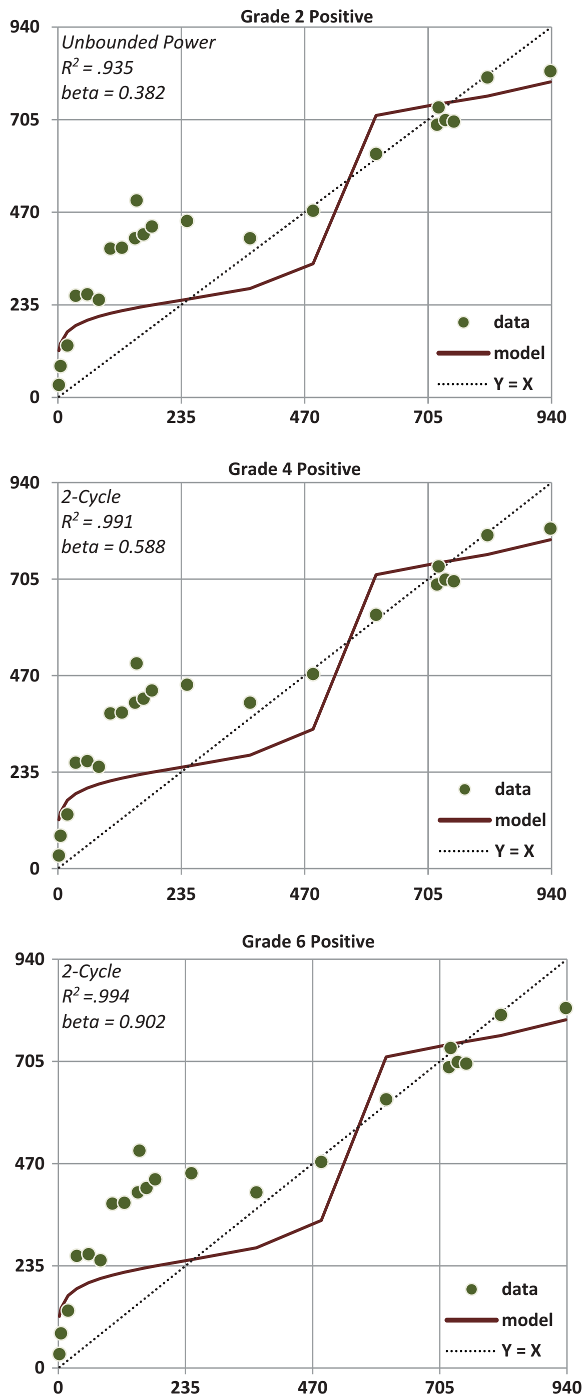

FIGURE 2 Best-fitting proportion shift for median positive number line estimates for second, fourth, and sixth graders.

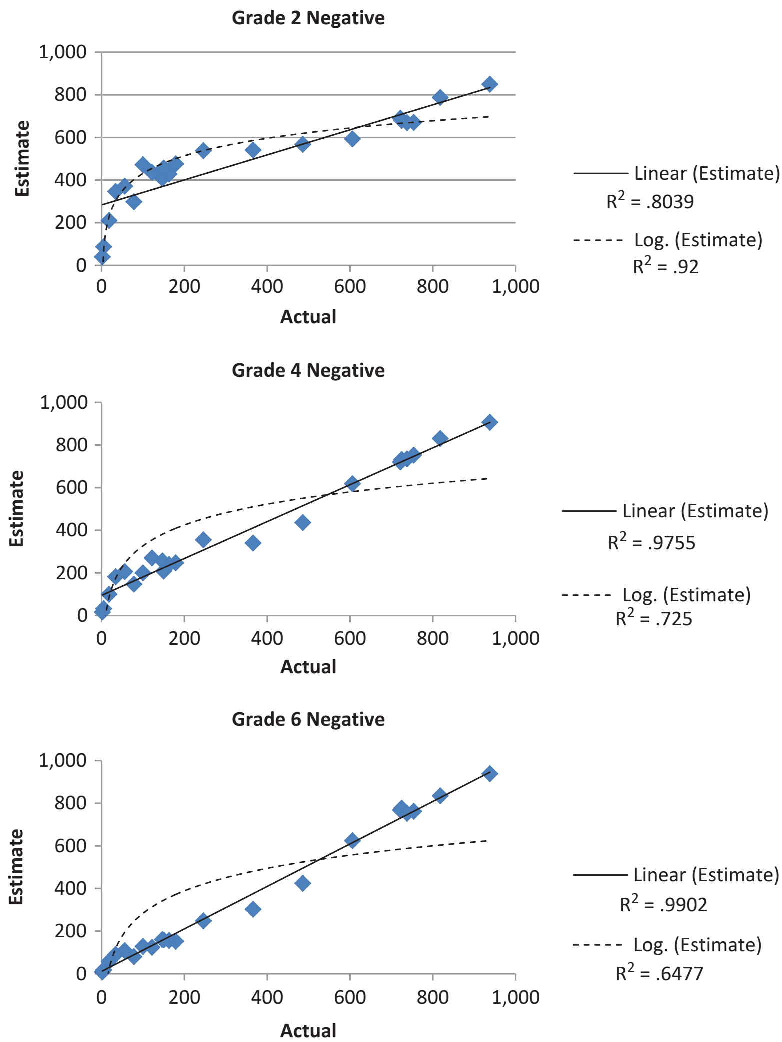

FIGURE 3 Best-fitting equations for median negative number line estimates for second, fourth, and sixth graders. Log. = logarithmic.

each grade level on the negative number estimates, the logarithmic function provided the best fit for 61.1% of Grade 2 students and the linear function provided the best fit for 38.9% in Grade 2 students. In Grade 4, the linear function provided the best fit for 79.3% of students and the logarithmic provided the best fit for 20.7%. With students in Grade 6, the linear function provided the best fit for 96.6% of students and the logarithmic only provided the best fit for 3.4%. Paired-sample *t*-tests indicated again that there was no difference in the fit of the linear function versus the fit of the logarithmic function for second graders (mean $R^2_{lin} = .55$, $SE = .04$, vs. mean $R^2_{log} = .59$, $SE = .03$), $t(71) = –1.53$, $p > .05$. However, for fourth graders, the fit on the linear function was a significantly better fit than the logarithmic function (mean R^2_{lin} = .82, SE = .02, vs. mean R^2_{log} = .65, SE = .02), $t(81) = 7.86$, $p < .001$, $d = 0.91$; and the estimates of sixth graders followed the same pattern (mean R^2_{lin} = .95, SE = .01, vs. mean R^2_{log} = .64, SE = .01), $t(86) = 27.00$, $p < .001$, $d = 3.1$. A one-way ANOVA indicated that the fit of the linear function to individual students' negative number estimates again increased with grade level, $F(2, 238) = 66.45$, $p < .001$. Games-Howell post-hoc tests indicated significant differences between all grade levels ($p < .01$).

We also examined the alternative proportion judgment approach and again considered unbounded power, one-cycle, and two-cycle models for students' estimation of negative values on the negative number line. For Grade 2 students, we again found the unbounded power function provided the best fit (R^2 = .94) and the two-cycle model was most appropriate for Grade 4 (R^2 = .99) and Grade 6 (R^2 = .99). Again, graphical representations of these models are presented in Figure 4.

Positive and Negative Values

Students' numerical estimation moved from logarithmic to linear as grade level increased with both positive and negative values. The greatest gains in estimation precision were from second to fourth grade, moving from 19.8% absolute error to 9.1% error for positive values and from 23.3% to 13.8% for negative numbers. Students were the most precise in numerical estimation in sixth grade with the lowest amount of absolute error (5.5% for positive values and 6.5% for negative values). However, there is a significant difference between the absolute error rates for positive and negative values, $t(239) = 4.44$, $p < .001$, with students having higher error rates for negative numbers. This pattern of difference is the same at each grade level and is statistically significant (see Table 1).

Furthermore, we examined other values of fit across positive and negative values, including the percentage of students who had logarithmic as a better fit in Grade 2 and linear as a better fit in Grades 4 and 6 with both positive and negative number lines. Using related-samples McNemar tests, we compared the percentage of Grade 2 students who had a logarithmic best fit for positive values (55.6%) and for negative values (61.1%), and at Grades 4, we compared the percentage of linear fit between positive values (81.7%) and negative values (79.3%); we found no significant differences. At Grade 6, the percentage of linear fit was 97% for both positive and negative values. Lastly, we examined the relationship between performance on positive and negative number lines across logarithmic and linear fits at the individual level. Correlations were moderate to strong and ranged from .40 to .81 when examined for the entire group and also by grade level indicating that students do tend to represent numbers similarly for positive and negative number lines.

Patterns of fit for the proportion judgment shift were similar for both negative and positive values with Grade 2 students having the best fit as determined by AICc (see Tables 2 and 3) using an unbounded power curve and Grade 4 and Grade 6 with better fit using the two-cycle curve. The beta (β) parameter was also calculated as a measure of bias; this parameter becomes larger with increased accuracy. Accuracy in the positive condition was better than in the negative condition and increased with age as can be seen by the beta values associated with the proportion judgment models. In the positive condition, the beta at Grade 2 was .38 and increased to .59 at Grade 4 and finally to .90 at Grade 6. For the negative condition, the beta value was .34

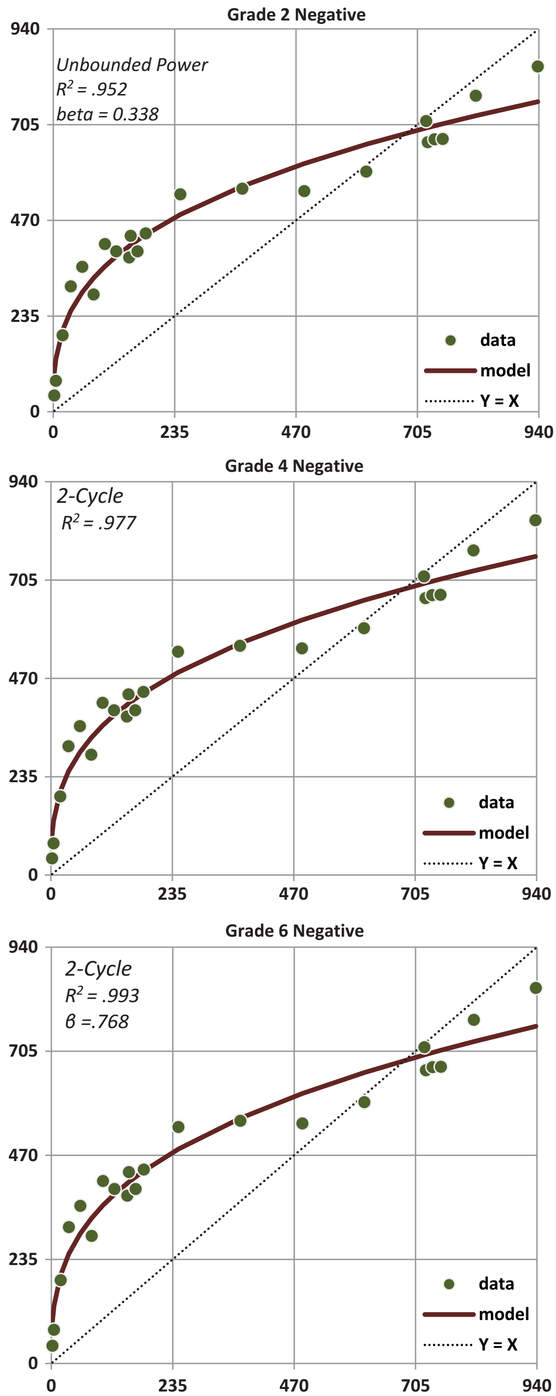

FIGURE 4 Best-fitting proportion shift for median negative number line estimates for second, fourth, and sixth graders.

TABLE 1
Comparison of Absolute Error Between Conditions

Grade Level	*Positive Numbers* Mean (SD)	*Negative Numbers* Mean (SD)	*t* Value
Second Grade (n = 72)	0.20 (0.11)	0.23 (.012)	–2.21*
Fourth Grade (n = 82)	0.09 (0.05)	0.14 (0.13)	–3.52**
Sixth Grade (n = 87)	0.05 (0.04)	0.06 (0.04)	–2.58*
Overall (n = 241)	0.11 (0.09)	0.14 (0.12)	–4.44**

* p < .05. ** p < .01.

TABLE 2
Estimates of Relative Support for Each Model, Positive Values

		Log-to-Linear Shift		*Proportion Judgment*		
		Logarithmic	*Linear*	*Unbounded Power*	*One Cycle*	*Two Cycle*
Grade 2	R^2	.794	.862	.935	.859	.687
	β			0.382	0.404	0.211
	AICc	208.156	199.382	*182.947*	197.403	214.966
	ΔAICc	25.209	16.435		14.456	32.019
Grade 4	R^2	.534	.988	.813	.986	.991
	β			0.402	0.821	0.588
	AICc	235.813	156.104	215.767	155.658	*146.691*
	ΔAICc	89.122	**9.413**	69.076	8.967	
Grade 6	R^2	.461	.994	.786	.994	.994
	β			0.461	1.041	0.902
	AICc	243.470	143.914	223.174	140.702	*140.331*
	ΔAICc	103.139	**3.583**	82.843	**0.371**	

Note. ΔAICc is the difference in Akaike information criterion (AICc) values compared with the preferred model appearing in italics. ΔAICc within 1 to 2 have substantial support and ΔAICc > 10 have no support (Burnham & Anderson, 2002). ΔAICc < 10 are in bold.

at Grade 2 and increased to .45 at Grade 4 and .77 at Grade 6. Similar to Slusser and colleagues (2013), we also found a tendency of overcorrection in older students (Grade 6) with beta values greater than 1 in the one-cycle models for both positive and negative values; however the AICc values suggest that the two-cycle model is also a valid model of explanation for this age group.

These proportion models address how participants approached completion of the number-line task. For example, an unbounded model is one in which no reference points are used, the one-cycle model has two references points (i.e., the two endpoints), and a two-cycle model has three reference points (i.e., the two endpoints and an inferred midpoint). An unbounded model offers the least systematic way to approach numbers on a number line as there are no reference points, leading to a high percentage of error. Error in judgment decreases with a one-cycle model as there are two reference points used to make more patterned guesses when placing a number on the number line. The percent of error decreases even further when using a two-cycle model as

TABLE 3
Estimates of Relative Support for Each Model, Negative Values

		Log-to-Linear Shift		*Proportion Judgment*		
		Logarithmic	*Linear*	*Unbounded Power*	*One Cycle*	*Two Cycle*
Grade 2	R^2	.863	.845	.952	.844	.484
	β			0.338	0.352	0.201
	AICc	195.950	198.623	*172.777*	196.315	222.674
	ΔAICc	23.174	25.846		23.538	49.897
Grade 4	R^2	.579	.977	.864	.965	.977
	β			0.417	0.731	0.447
	AICc	232.841	169.196	208.023	175.828	*166.239*
	ΔAICc	66.602	**2.957**	41.784	**9.589**	
Grade 6	R^2	.474	.990	.776	.989	.993
	β			0.423	1.029	0.768
	AICc	243.012	155.375	224.258	154.644	*146.344*
	ΔAICc	96.668	**9.031**	77.914	**8.300**	

Note. ΔAICc is the difference in Akaike information criterion (AICc) values compared with the preferred model appearing in italics. ΔAICc within 1 to 2 have substantial support and ΔAICc > 10 have no support (Burnham & Anderson, 2002). ΔAICc < 10 are in bold.

both endpoints and a midpoint are used to make systematic judgments about numbers on a number line.

Logarithmic-to-Linear Shift and Proportion Judgment

The relative ability of the logarithmic-to-linear shift pattern or the proportion judgment to explain the mental representation of positive versus negative numbers is no different. Similar patterns were observed for each grade level in both approaches to numeric representation, as can be seen by the estimates of fit in Table 2 for positive values and Table 3 for negative values. At Grade 2 in both positive and negative estimates, the unbounded power curve was the uncontested better fit, while at Grade 4 and Grade 6, there could be some debate over the two-cycle model versus the linear fit pattern as the difference in AICc is less than 10 (using the same criteria as Slusser et al., 2013). Furthermore, there is not a clear conclusion at Grade 6 between the one-cycle and two-cycle models in both positive and negative values.

DISCUSSION

Our findings replicate those of Siegler and colleagues (e.g., Booth & Siegler, 2006) for positive numbers; specifically, we found evidence of a shift from logarithmic to linear representations for positive numbers between fourth and sixth grade (0–1,000 scale). More interestingly, we found a similar pattern of results for negative numbers. While the second graders' estimations were best fit by a logarithmic function for negative number lines, the fourth and sixth graders' estimations were best fit by a linear function.

It is possible that these findings suggest that children's mental representations of negative numbers develop in parallel to their representations of positive numbers. As evidenced by the data, we observed the same shift from a logarithmic to linear representation for negative numbers as we did with the positive numbers suggesting that children could be representing both positive and negative numbers in a combined representation. However, this explanation seems unlikely as accuracy was lower for the negative numbers than the positive numbers, despite the similar logarithmic-to-linear shift in mental representation for both conditions. One explanation for this difference in accuracy could be that children are processing negative numbers differently than they are processing positive numbers. Children may store magnitude and polarity separately, similarly to adults (Tzelgov et al., 2009). Furthermore, it is possible to solve these number-line estimation tasks by ignoring the negative number sign on both the target number and the number line and thus treating the problem as positive numbers. The only difference between this strategy and the standard positive number-line estimation task is that the number line runs from left to right instead of right to left in the standard version. Again, this strategy is similar to that used by adults and transposes the number line to place the mark as if it were a positive number line (Karjcsi & Igacs, 2010; Varma & Schwartz, 2011). Thus, by ignoring the magnitude properties, children may show similar patterns of representation for both the positive and negative numbers. However, this explanation still fails to address the difference in accuracy across these conditions.

A second explanation is that children are not as familiar with negative numbers because they do not receive formal education with negative numbers until fifth or sixth grade. Ebersbach, Luwel, Frick, Onghena, and Verschaffel (2008) found that children's familiarity with numbers (positive only) related to their mental representations. Because children are not as familiar with negative numbers, it could reduce their accuracy in this condition. Unfortunately, the current study does not allow us to distinguish between these alternatives, as we cannot determine the strategies that children used for solving these problems. However, these findings do open up many fascinating avenues of research to better understand how children are mentally representing negative numbers. Specifically, future studies should follow up with fourth graders as these students have not been formally taught about negative numbers yet and show the same pattern of representation as the sixth graders who do have formal training in this domain. This finding suggests that formal instruction with negative numbers may not be necessary for forming a mental representation of negative numbers, but further research is needed before any firm conclusions about this implication can be made. Perhaps by better understanding the strategies that children are using in this task, we may better be able to interpret these fascinating and novel findings. Additionally, because of the established relationship of estimation tasks to math achievement (Booth & Siegler, 2006), understanding the development of numerical representation of negative numbers could impact when and how we teach children about negative numbers and related concepts.

One avenue for future research is to investigate how the presentation format affects students' performance on these tasks. In the current study, we gave students two separate lines (one for positive numbers and one for negative numbers) to more easily replicate findings from previous studies (e.g., Opfer & Siegler, 2007). However, it is possible that students were less likely to tap into any existing mental representation of negative numbers (if they have such a representation) because the format of the lines did not support or necessitate the use of negative numbers to successfully complete the task. For example, providing students with a single line that contains both the positive and negative numbers on a single scale could facilitate the use of students' representation of negative numbers. Future studies should address how the presentation format affects the processing of both positive and negative numbers.

Negative numbers are not taught in U.S. schools until fifth or sixth grade, so most of these children have no formal training with negative numbers. Understanding how children are solving these negative number tasks is important and should be a focus of future research. Piatt, Coret, Volden, and Bisanz (2012) presented some interesting research in which they categorized strategies that children used when completing number-line estimation tasks. For example, they looked to see how children used different anchor points on the line (e.g., endpoints, midpoint) and what kind of adjustments they made from those endpoints (e.g., How far away from the anchor point should a given target be?) to classify different strategies that children used. A simple strategy would be using the anchor points without making any adjustment. A more complicated strategy would be to segment the line and make estimations. Their most sophisticated strategy required making proportional adjustments from various endpoints to estimate where the target number should be. They found that children used these strategies with varying frequency depending on which target number was presented (Piatt et al., 2012). Of course, the number lines were positive number lines in their study, but it seems reasonable that a similar approach could be taken with the current findings from the negative number lines. We can make a course estimate of the strategies that children used based on the proportion judgment analyses conducted. The fact that the older children (Grades 4 and 6) were best fit by the two-cycle model suggests that these children were using both the endpoints and the midpoint to make numerical estimations. This finding was true for both the positive and negative numbers, suggesting that children use similar strategies for both. One avenue for further exploration would be a more detailed analysis of the various strategies that children are using in these tasks and how that may or may not differ for positive and negative numbers. This analysis could include a method similar to that of Piatt et al. (2012) or an interviewing task where children are explicitly asked to describe the strategies they use to complete the task. Our findings show that the logarithmic-to-linear shift occurs in parallel to the shift in proportion model judgments and that this shift happens as students advance through school and receive more formal training in math estimation. Therefore, older students are better able to make accurate judgments concerning number location on a number line using a pattern of either two reference points (i.e., one-cycle) or three reference points (i.e., two-cycle). This implication offers educators a systematic way to teach number sense to younger students by approaching this task through the use of reference points to determine patterns. Teaching younger students to use reference points should foster more accurate judgments about number placement on a number line, thereby leading to increased numerical literacy in younger students, which could lead to higher math achievement.

Although research in the area of numerical representations has moved forward in considering individual factors that may contribute to children's mental representation of numbers using positive values (Friso-van den Bos et al., 2014), the data from this study provide the first evidence of how negative number mental representations compare to positive number representations of elementary-age children. While theoretical explanations and data with adults are conflicting as to how adults represent negative numbers, the theories and data to explain the development of this construct are even sparser. Although it can be assumed that children's abilities to make judgments on numerical estimation tasks should improve during development, the data with negative numbers has been lacking. Plenty of evidence demonstrates that children's estimation of positive numbers becomes more accurate across ages (Booth & Siegler, 2006; Siegler & Opfer, 2003; Siegler et al., 2009), but studies have never addressed this same question with negative numbers. We found that children's representation of negative numbers becomes more linear with development, but furthermore, it also becomes more accurate. Although the exact strategies that children use to solve these problems remain unknown, we hope that these results will spur new research into better understanding how children solve negative number-line tasks and how they mentally represent negative numbers. Furthermore, if we can better understand how children are representing and processing negative numbers, we may be able to find teaching methods that better align with children's developmental trajectory of this concept, thereby leading to improved teaching methods, such as teaching reference points, and higher mathematics competency among elementary students.

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